THREE-PHASE SYSTEMS

The three-phase systems are used for

- generation

- transmission

- distribution

of electrical power because

- The three-phase generators are less bulky and have a lower weight than the other electrical systems single-phase and d.c.

- The three-phase electrical lines have a lower weight than the other match all electrical parameters.

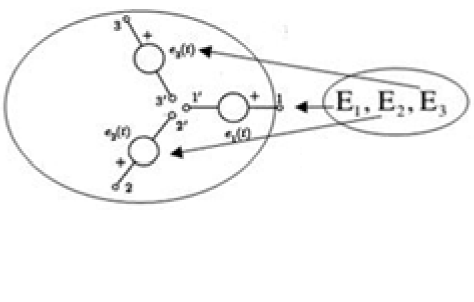
From the electrical point of view, a three-phase generator is composed of three single-phase generators with voltages of equal amplitudes and phase differences of 120°, according to the following relationships:

$$\boldsymbol{E}_1 = E_M e^{j0}$$

$$\boldsymbol{E}_2 = E_M e^{-j\frac{2}{3}\pi}$$

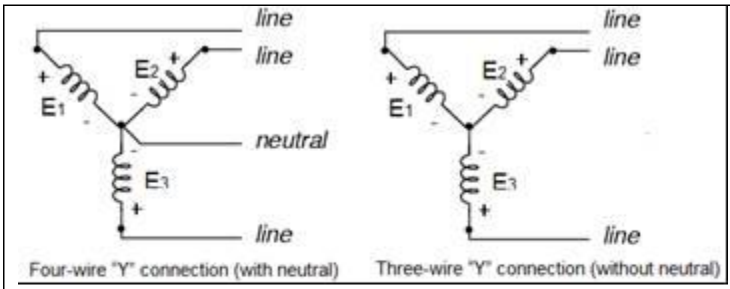
$$\boldsymbol{E}_3 = E_M e^{-j\frac{4}{3}\pi} = E_M e^{j\frac{2}{3}\pi}$$

The *phase* voltage is the voltage of each of the single phase generator

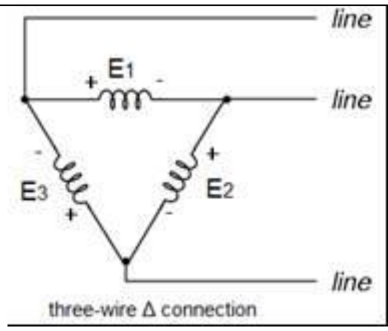


These generator can be connected in different ways:

- Wye or Star connection (Y): the three voltage sources are connected to a common point with or without neutral wire;
- delta (Δ) connection



Four-wire "Y" connection (with neutral) Three-wire "Y" connection (without neutral)



three-wire Δ connection

The three-phase systems are classified according to the sets of three electromotive forces (emf) and currents.

A "**Symmetric 3-phase set**" is a three phase system where the set of three sinusoidal voltages satisfy these requirements:

- All three voltages have the same amplitude
- All three voltages have the same frequency
- All three voltages are 120° in phase

So is called **symmetric** a system where the emfs

$$\boldsymbol{E}_1 + \boldsymbol{E}_2 + \boldsymbol{E}_3 = \boldsymbol{0}$$

A "**balanced 3-phase set**" is a three phase system where the set of three sinusoidal currents satisfy these requirements:

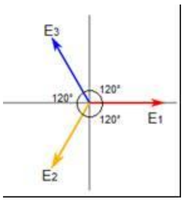
- All three currents have the same amplitude
- All three currents have the same frequency
- All three currentes are 120° in phase

So is called **balanced** a three phase system where the currents

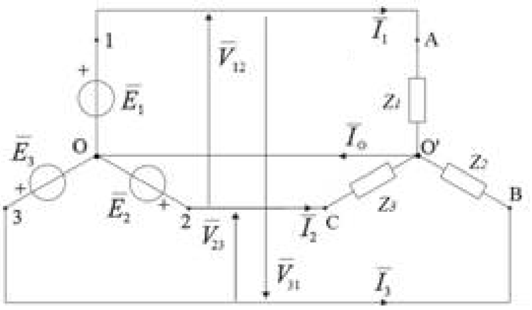
$$\boldsymbol{I}_1 + \boldsymbol{I}_2 + \boldsymbol{I}_3 = 0$$

If the load (Wye or Delta) has equal impedances, the system is **Balanced**

If the load (Wye or Delta) hasn't equal impedances, the system is **Unbalanced**



SYMMETRIC AND BALANCED THREE-PHASE SYSTEMS (BALANCED WYE-WYE CONNECTION)



Source current = Line currents = Load currents

$$\boldsymbol{I_1} = \frac{\boldsymbol{E_1}}{\boldsymbol{Z}}, \quad \boldsymbol{I_2} = \frac{\boldsymbol{E_2}}{\boldsymbol{Z}}, \quad \boldsymbol{I_3} = \frac{\boldsymbol{E_3}}{\boldsymbol{Z}}$$

Neutral currents is zero (so the neutral wire is unnecessary)

$$\boldsymbol{I_0} = -(\boldsymbol{I_1} + \boldsymbol{I_2} + \boldsymbol{I_3}) = -\frac{1}{\boldsymbol{Z}}(\boldsymbol{E_1} + \boldsymbol{E_2} + \boldsymbol{E_3}) = \boldsymbol{0}$$

Because the three phase system is symmetric, it is

$$(\boldsymbol{E_1} + \boldsymbol{E_2} + \boldsymbol{E_3}) = 0$$

The line-to-line voltages (**V**$_{12}$, **V**$_{23}$, **V**$_{31}$) can be expressed in terms of the line-to-neutral voltages (**E**$_1$, **E**$_2$, **E**$_3$)

$$\boldsymbol{V_{12}} = \boldsymbol{E_1} - \boldsymbol{E_2} = E_M - E_M e^{-j120^\circ} = E_M\left(1 + 0{,}5 + j\frac{\sqrt{3}}{2}\right) =$$

$$E_M\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}E_M\left(\frac{\sqrt{3}}{2} + j0{,}5\right) = \sqrt{3}E_M e^{j30^\circ} = \sqrt{3}E_1 e^{j30^\circ}$$

$$|\boldsymbol{V_{12}}| = \sqrt{3}|\boldsymbol{E_1}| \angle \boldsymbol{V_{12}} = \angle \boldsymbol{E_1} + 30^\circ$$

$$|\boldsymbol{V_{23}}| = \sqrt{3}|\boldsymbol{E_2}| \angle \boldsymbol{V_{23}} = \angle \boldsymbol{E_2} + 30^\circ$$

$|\boldsymbol{V}_{31}| = \sqrt{3}|\boldsymbol{E}_3| \angle\boldsymbol{V}_{31} = \angle\boldsymbol{E}_3 + 30^\circ$

If we compare the line-to-neutral voltages with the line-to-line voltages, we find the following relationships:

Line to line voltages in terms of line to neutral voltages

$|\boldsymbol{V}_{12}| = \sqrt{3}|\boldsymbol{E}_1| \angle\boldsymbol{V}_{12} = \angle\boldsymbol{E}_1 + 30^\circ$

$|\boldsymbol{V}_{23}| = \sqrt{3}|\boldsymbol{E}_2| \angle\boldsymbol{V}_{23} = \angle\boldsymbol{E}_2 + 30^\circ$

$|\boldsymbol{V}_{31}| = \sqrt{3}|\boldsymbol{E}_3| \angle\boldsymbol{V}_{31} = \angle\boldsymbol{E}_3 + 30^\circ$

In any Symmetric and balanced three-phase systems (symmetric voltages and balanced load impedances), the resulting currents are balanced. This way, there is necessary to analyze all three phases. We may analyze one phase to determine its current, and deduce the currents in the other phases based on a simple balanced phase shift (120° phase difference between any two line currents).

Symmetric and balanced three-phase systems (DELTA-DELTA CONNECTION)

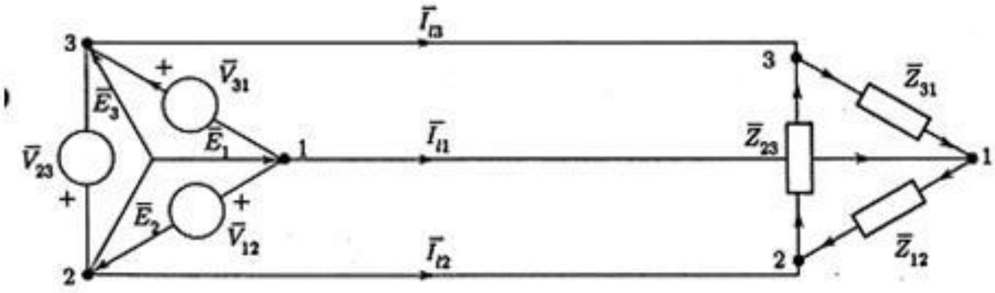


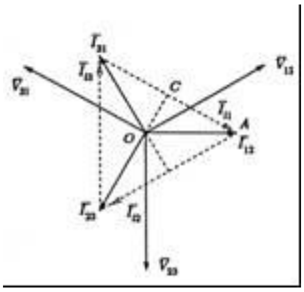
Fig. 4.8: Sistema simmetrico di tensioni collegato a triangolo con un carico regolare pure esso collegato a triangolo.

The Line currents are different than the load currents

Line currents ≠ load currents

$\mathbf{I}_1 = \mathbf{I}_{12} - \mathbf{I}_{31}$ $\quad\quad$ $\mathbf{I}_2 = \mathbf{I}_{23} - \mathbf{I}_{12}$ $\quad\quad$ $\mathbf{I}_3 = \mathbf{I}_{31} - \mathbf{I}_{23}$

$\mathbf{Z}_{12} = \mathbf{Z}_{23} = \mathbf{Z}_{31} = \mathbf{Z}$ inductive resistive load



$$\boldsymbol{I}_{12} = \frac{\boldsymbol{V}_{12}}{\boldsymbol{Z}}$$

$$\boldsymbol{I}_{23} = \frac{\boldsymbol{V}_{23}}{\boldsymbol{Z}} = \frac{V_{12}}{Z} e^{-j120^\circ} = I_{12} e^{-j120^\circ}$$

$$\boldsymbol{I}_{31} = \frac{\boldsymbol{V}_{31}}{\boldsymbol{Z}} = \frac{V_{12}}{Z} e^{-j240^\circ} = I_{12} e^{-j240^\circ}$$

relationship between the line currents and phase:

$$\boldsymbol{I}_1 = \boldsymbol{I}_{12} - \boldsymbol{I}_{31} = I_{12} - I_{12}e^{j120^\circ} = I_{12}\left(1 + 0{,}5 - J\frac{\sqrt{3}}{2}\right) =$$

$$I_{12}\left(\frac{3}{2} - J\frac{\sqrt{3}}{2}\right) = \sqrt{3}I_{12}\left(\frac{\sqrt{3}}{2} - J0{,}5\right) = \sqrt{3}I_{12}e^{-j30^\circ}$$

$$|I_1| = \sqrt{3}|\boldsymbol{I}_{12}| \angle \boldsymbol{I}_1 = \angle \boldsymbol{I}_{12} - 30^\circ$$

$$|\boldsymbol{I}_2| = \sqrt{3}|\boldsymbol{I}_{23}| \angle \boldsymbol{I}_2 = \angle \boldsymbol{I}_{23} - 30^\circ$$

$$|\boldsymbol{I}_3| = \sqrt{3}|\boldsymbol{I}_{31}| \angle \boldsymbol{I}_3 = \angle \boldsymbol{I}_{31} - 30^\circ$$

Symmetric and balanced three-phase systems (WYE-DELTA CONNECTION)

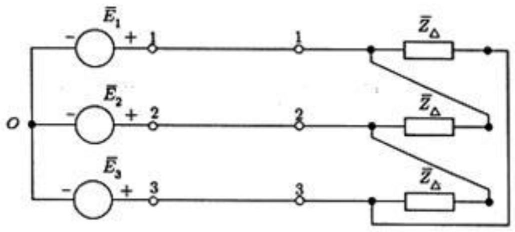
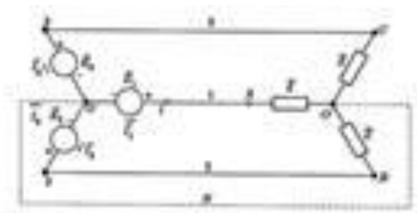


Fig. 4.10: Fasi del generatore collegate a stella e carico collegato a triangolo.

We can use the Wye-Delta transformation to get the case of wye load already studied

$$\boldsymbol{Z}_Y = \frac{\boldsymbol{Z}_\Delta}{3} = \boldsymbol{Z}$$



Symmetric and balanced three-phase systems (DELTA-WYE CONNECTION)

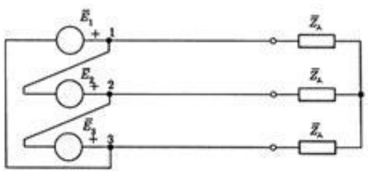
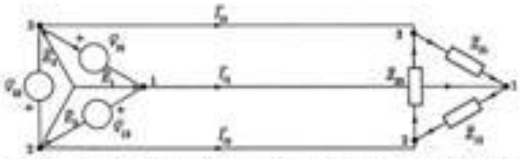
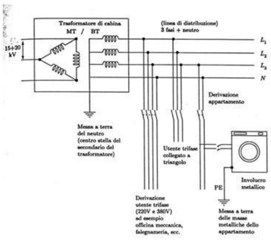


Fig. 4.12: Fasi del generatore collegate a triangolo e carico collegato a stella.

We can use the Delta-Wye transformation to get the case of wye sources already studied: $\mathbf{Z}_{12} = \mathbf{Z}_{23} = \mathbf{Z}_{31} = \mathbf{Z} = 3\mathbf{Z}_Y$

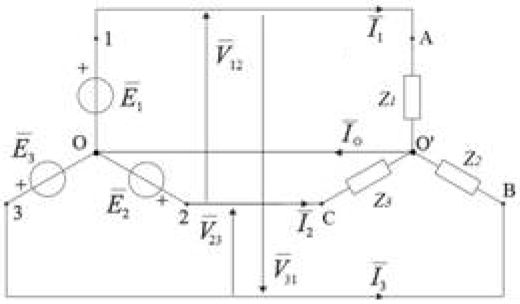


Sistema di distribuzione in bassa tensione di tipo T.T.



Carichi irregolari con sistema trifase squilibrato nella corrente, ma simmetrico nella tensioni (supponendo trascurabili le cadute di tensione sulle fasi del generatore)

SYMMETRIC AND UNBALANCED THREE-PHASE SYSTEMS WITH NEUTRAL WIRE (WYE-WYE CONNECTION)



E$_1$, **E**$_2$, **E**$_3$: the set of three symmetric voltages

Z$_1$ $\neq$ **Z**$_2$ $\neq$ **Z**$_3$: the set of three unbalanced impedances

The presence of the neutral wire ensures that O and O 'are at the same potential and then we have three independent phases:

$$\boldsymbol{I}_1 = \frac{\boldsymbol{E}_1}{\boldsymbol{Z}_1} \qquad \boldsymbol{I}_2 = \frac{\boldsymbol{E}_2}{\boldsymbol{Z}_2} \qquad \boldsymbol{I}_3 = \frac{\boldsymbol{E}_3}{\boldsymbol{Z}_3}$$

In neutral wire circulates a current equal to

$$\boldsymbol{I}_N = -(\boldsymbol{I}_1 + \boldsymbol{I}_2 + \boldsymbol{I}_3) \neq 0$$

SYMMETRIC AND UNBALANCED THREE-PHASE SYSTEMS WITHOUT NEUTRAL WIRE (WYE-WYE CONNECTION)

$\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$ the set of three symmetric voltages

$\mathbf{Z}_1 \neq \mathbf{Z}_2 \neq \mathbf{Z}_3$: the set of three unbalanced impedances

The absence of the neutral wire introduces an **electric potential difference** between O and O ': $\mathbf{E}_{O'O}$

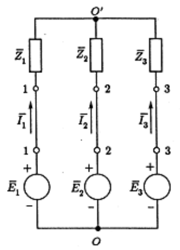


Fig. 4.19: Carico irregolare in collegamento stella-spuria.

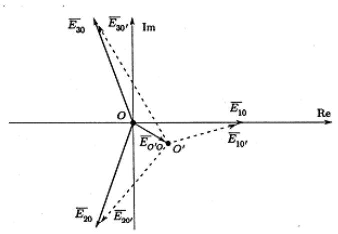


Fig. 4.20: Diagramma fasoriale nel caso di carico in configurazione stella spuria.

$$\begin{cases} \boldsymbol{E}_{10'} = \boldsymbol{E}_1 - \boldsymbol{E}_{O'O} \\ \boldsymbol{E}_{20'} = \boldsymbol{E}_2 - \boldsymbol{E}_{O'O} \\ \boldsymbol{E}_{30'} = \boldsymbol{E}_3 - \boldsymbol{E}_{O'O} \end{cases}$$

$$\boldsymbol{I}_1 = \frac{\boldsymbol{E}_{10'}}{\boldsymbol{Z}_1}, \quad \boldsymbol{I}_2 = \frac{\boldsymbol{E}_{20'}}{\boldsymbol{Z}_2}, \quad \boldsymbol{I}_3 = \frac{\boldsymbol{E}_{30'}}{\boldsymbol{Z}_3}$$

$$\begin{cases} \boldsymbol{E}_1 - \boldsymbol{E}_3 = (\boldsymbol{Z}_1 + \boldsymbol{Z}_3)\boldsymbol{I}_1 + \boldsymbol{Z}_3\boldsymbol{I}_2 \\ \boldsymbol{E}_2 - \boldsymbol{E}_3 = \boldsymbol{Z}_3\boldsymbol{I}_1 + (\boldsymbol{Z}_2 + \boldsymbol{Z}_3)\boldsymbol{I}_2 \\ \boldsymbol{I}_3 = -(\boldsymbol{I}_1 + \boldsymbol{I}_2) \end{cases}$$

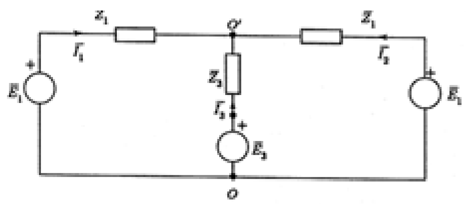


Fig. 4.21: Analisi di carico irregolare collegato a stella.

$$\begin{cases} \boldsymbol{I}_1 = \dfrac{\boldsymbol{Z}_2(\boldsymbol{E}_1 - \boldsymbol{E}_3) + \boldsymbol{Z}_3(\boldsymbol{E}_1 - \boldsymbol{E}_2)}{\boldsymbol{Z}_1\boldsymbol{Z}_2 + \boldsymbol{Z}_1\boldsymbol{Z}_3 + \boldsymbol{Z}_2\boldsymbol{Z}_3} \\ \boldsymbol{I}_2 = \dfrac{\boldsymbol{Z}_1(\boldsymbol{E}_2 - \boldsymbol{E}_3) + \boldsymbol{Z}_3(\boldsymbol{E}_2 - \boldsymbol{E}_1)}{\boldsymbol{Z}_1\boldsymbol{Z}_2 + \boldsymbol{Z}_1\boldsymbol{Z}_3 + \boldsymbol{Z}_2\boldsymbol{Z}_3} \\ \boldsymbol{I}_3 = \dfrac{\boldsymbol{Z}_2(\boldsymbol{E}_3 - \boldsymbol{E}_1) + \boldsymbol{Z}_3(\boldsymbol{E}_2 - \boldsymbol{E}_1)}{\boldsymbol{Z}_1\boldsymbol{Z}_2 + \boldsymbol{Z}_1\boldsymbol{Z}_3 + \boldsymbol{Z}_2\boldsymbol{Z}_3} \end{cases}$$

$$\boldsymbol{E}_{O'O} = \frac{\boldsymbol{E}_1\boldsymbol{Y}_1 + \boldsymbol{E}_2\boldsymbol{Y}_2 + \boldsymbol{E}_3\boldsymbol{Y}_3}{\boldsymbol{Y}_1 + \boldsymbol{Y}_2 + \boldsymbol{Y}_3}$$

SYMMETRIC AND UNBALANCED THREE-PHASE SYSTEMS (DELTA-DELTA CONNECTION)

$\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$ the set of three symmetric voltages

$\mathbf{Z}_1 \neq \mathbf{Z}_2 \neq \mathbf{Z}_3$: the set of three unbalanced impedances

$$\boldsymbol{I}_{12} = \frac{\boldsymbol{V}_{12}}{\boldsymbol{Z}_{12}}, \qquad \boldsymbol{I}_{23} = \frac{\boldsymbol{V}_{23}}{\boldsymbol{Z}_{23}}, \qquad \boldsymbol{I}_{31} = \frac{\boldsymbol{V}_{31}}{\boldsymbol{Z}_{31}}$$

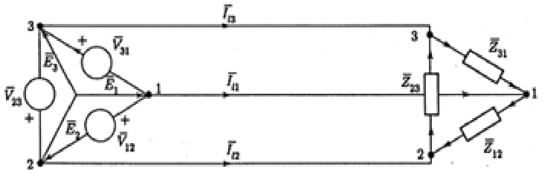


Fig. 4.8: Sistema simmetrico di tensioni collegato a triangolo con un carico regolare pure esso collegato a triangolo.

NOT SYMMETRIC AND UNBALANCED THREE-PHASE SYSTEMS

$\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$ the set of three symmetric voltages

$\mathbf{Z}_1 \neq \mathbf{Z}_2 \neq \mathbf{Z}_3$: the set of three unbalanced impedances (Delta load)

DELTA LOAD

$$\boldsymbol{I}_{12} = \frac{\boldsymbol{V}_{12}}{\boldsymbol{Z}_{12}}, \qquad \boldsymbol{I}_{23} = \frac{\boldsymbol{V}_{23}}{\boldsymbol{Z}_{23}}, \qquad \boldsymbol{I}_{31} = \frac{\boldsymbol{V}_{31}}{\boldsymbol{Z}_{31}}$$

$$\boldsymbol{I}_1 = \boldsymbol{I}_{12} - \boldsymbol{I}_{31}\ , \qquad \boldsymbol{I}_2 = \boldsymbol{I}_{23} - \boldsymbol{I}_{12}\ , \quad \boldsymbol{I}_3 = \boldsymbol{I}_{31} - \boldsymbol{I}_{23}$$

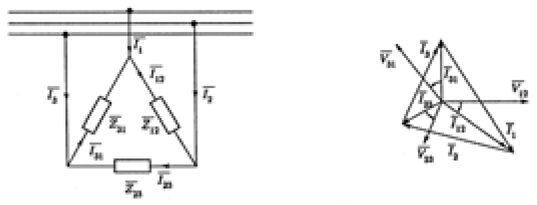


Fig. 4.23: Sistema disimmetrico e carico equilibrato.

WYE LOAD

as above after having transformed the wye load into delta load

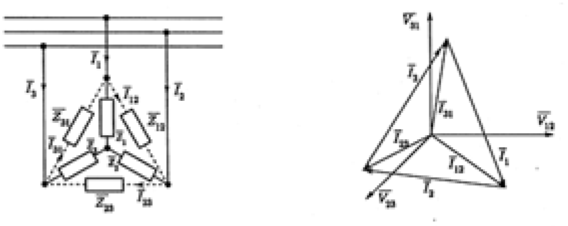


Fig. 4.24: Sistema disimmetrico con carico non regolare collegato a stella.

$$\boldsymbol{Z}_{12} = \frac{\boldsymbol{Z}_1\boldsymbol{Z}_2 + \boldsymbol{Z}_1\boldsymbol{Z}_3 + \boldsymbol{Z}_2\boldsymbol{Z}_3}{\boldsymbol{Z}_3}$$

$$\boldsymbol{Z}_{23} = \frac{\boldsymbol{Z}_1\boldsymbol{Z}_2 + \boldsymbol{Z}_1\boldsymbol{Z}_3 + \boldsymbol{Z}_2\boldsymbol{Z}_3}{\boldsymbol{Z}_1}$$

$$\boldsymbol{Z}_{31} = \frac{\boldsymbol{Z}_1\boldsymbol{Z}_2 + \boldsymbol{Z}_1\boldsymbol{Z}_3 + \boldsymbol{Z}_2\boldsymbol{Z}_3}{\boldsymbol{Z}_2}$$

POWER IN THREE PHASE SYSTEM

The total power of the system is given by the sum of the three powers of each phase

A) Symmetric and balanced system

Voltage on the load $e(t) = Ecos(\omega t)$ current $i(t) = Icos(\omega t - \varphi)$

φ = phase angle between the current and the voltage

$$p(t) = p_1(t) + p_2(t) + p_3(t) =$$

$$= E\cos(\omega t)\, I\cos(\omega t - \varphi) + E\cos\left(\omega t - \frac{2}{3}\pi\right) I\cos\left(\omega t - \frac{2}{3}\pi - \varphi\right) +$$

$$+ E\cos\left(\omega t - \frac{4}{3}\pi\right) I\cos\left(\omega t - \frac{4}{3}\pi - \varphi\right) =$$

$$= \frac{1}{2}\left[EI\cos\varphi + EI\cos(2\omega t + \varphi) + EI\cos\varphi + EI\cos\left(2\omega t + \varphi - \frac{2}{3}\pi\right) +\right.$$

$$\left. + EI\cos\varphi + EI\cos\left(2\omega t + \varphi - \frac{4}{3}\pi\right)\right] =$$

$$= \frac{3}{2} EI\cos\varphi = 3E_{eff}I_{eff}\cos\varphi$$

The total power supplied to the load is constant (unlike transmission with three-phase generators

$$S = VI = |V|e^{j\varphi_v}\frac{|V|e^{j\varphi_v}}{|Z|e^{j\varphi}} = |V||I|e^{j\varphi} = S(cos\varphi + j\ sin\varphi) = P + jQ$$

S = apparent power P = active power Q = reactive power

Wye load	Delta load
$P = 3\|\boldsymbol{E}\|\|\boldsymbol{I}_L\|cos\varphi = \sqrt{3}\|\boldsymbol{V}\|\|\boldsymbol{I}_L\|cos\varphi$	$P = 3\|\boldsymbol{V}\|\|\boldsymbol{I}_f\|cos\varphi = \sqrt{3}\|\boldsymbol{V}\|\|\boldsymbol{I}_L\|cos\varphi$
$Q = 3\|\boldsymbol{E}\|\|\boldsymbol{I}_L\|sin\varphi = \sqrt{3}\|\boldsymbol{V}\|\|\boldsymbol{I}_L\|sin\varphi$	$Q = 3\|\boldsymbol{V}\|\|\boldsymbol{I}_f\|sin\varphi = \sqrt{3}\|\boldsymbol{V}\|\|\boldsymbol{I}_L\|sin\varphi$

The power factor (P.F) $P.F = cos\left[tan^{-1}(\frac{Q}{P})\right] = cos\varphi$

POWER IN THREE PHASE SYSTEM

The total power of the system is given by the sum of the three powers of each phase

A) Not Symmetric and unbalanced system

The total power supplied to the load isn't constant

$$p(t) = p_1(t) + p_2(t) + p_3(t) =$$

$$= E_1 \cos(\omega t) I_1 \cos(\omega t - \varphi_1) + E_2 \cos\left(\omega t - \frac{2}{3}\pi\right) I_2 \cos\left(\omega t - \frac{2}{3}\pi - \varphi_2\right) +$$

$$+ E_3 \cos\left(\omega t - \frac{4}{3}\pi\right) I_3 \cos\left(\omega t - \frac{4}{3}\pi - \varphi_3\right) =$$

$$= E_1 I_1 \cos\varphi_1 + E_1 I_1 \cos(2\omega t + \varphi_1) + E_2 I_2 \cos\varphi_2 + E_2 I_2 \cos\left(2\omega t + \varphi_2 - \frac{2}{3}\pi\right) +$$

$$+ E_3 I_3 \cos\varphi_3 + E_3 I_3 \cos\left(2\omega t + \varphi_3 - \frac{4}{3}\pi\right)$$

$$P = |\boldsymbol{E}_1|\,|\boldsymbol{I}_1| \cos\varphi_1 + |\boldsymbol{E}_2|\,|\boldsymbol{I}_2| \cos\varphi_2 + |\boldsymbol{E}_3|\,|\boldsymbol{I}_3| \cos\varphi_3$$

$$Q = |\boldsymbol{E}_1|\,|\boldsymbol{I}_1| \sin\varphi_1 + |\boldsymbol{E}_2|\,|\boldsymbol{I}_2| \sin\varphi_2 + |\boldsymbol{E}_3|\,|\boldsymbol{I}_3| \sin\varphi_3$$

$$P.F. = \cos\left[\tan^{-1}\left(\frac{Q}{P}\right)\right] \neq \cos\varphi_1,\ \cos\varphi_2, \cos\varphi_3$$